Spontaneous violation of the energy conditions

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ABSTRACT

A decade ago, it was shown that a wide class of scalar-tensor theories can pass very restrictive weak field tests of gravity and yet exhibit non-perturbative strong field deviations away from General Relativity. This phenomenon was called 'Spontaneous Scalarization' and causes the (Einstein frame) scalar field inside a neutron star to rapidly become inhomogeneous once the star's mass increases above some critical value. For a star whose mass is below the threshold, the field is instead nearly uniform (a state which minimises the star's energy) and the configuration is similar to the General Relativity one. Here, we show that the spontaneous scalarization phenomenon is linked to another strong field effect: a spontaneous violation of the weak energy condition.

Subject headings: gravitation, stars: neutron, relativity

1. Introduction

Scalar-tensor (ST) theories describe gravity as being mediated by both a metric g_{ab} and a scalar field Φ. The latter is coupled to the metric via a function $\omega(\Phi)$. These theories are fully conservative and only two parameterised post-Newtonian (PPN) parameters, γ and β, appear in the formalism. Currently, they are constrained to have the values $|\gamma - 1| \leq 0.0003$ and $|\beta - 1| \leq 0.002$ (see Will 1998 for details). For Brans-Dicke-like (BD) theories, the first of these inequalities implies that its coupling is today $\omega > 3300$. All predictions of BD theory differ from those of General Relativity (GR) to within a relative deviation of $\sim 1/\omega$. For other, more general, ST theories, the observational data only place limits on the behaviour of $\omega(\Phi)$ in the slow motion, weak field limit. However, strong field tests generally place weaker constraints on ST theories than those given above. Hence, it is possible to

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have a ST theory which satisfies all current constraints but shows strong field effects that are significantly different from GR. In this Letter we present the case for the appearance of a spontaneous violation of the energy conditions as one of such strong field effects.

2. Scalarization

Spontaneous scalarization was discovered by Damour & Esposito-Farese (1993, 1997) in scalar-tensor models of neutron stars. They found that for particular forms of the function $\omega(\Phi)$, see the Lagrangian density below, the Φ field inside a neutron star rapidly becomes inhomogeneous once the star's mass increases above some critical value. For a star whose mass is below this value, Φ is nearly homogeneous throughout the star (a state which minimises the star's energy), while for higher mass stars, the energy is minimised when the field has a large spatial variation. To exhibit these effects, $\omega(\Phi)$ must be such that its derivative with respect to Φ satisfies the inequality $\widetilde{\beta}_0 := 2\Phi_B(2\omega+3)^{-2}\,\mathrm{d}\omega/\mathrm{d}\Phi_B < -4$, where the subscript 'B' denotes cosmological or 'background' quantities evaluated far from strongly gravitating sources. Theories in which $\omega(\Phi)$ satisfies the above inequality may be arbitrarily close to GR in the weak field limit but yet significantly diverge from it in strong field regions. The scalarization effect becomes more pronounced as $\Phi_B \to 1$.

The Jordan frame (JF) action for the ST theories is

$$I = \int \mathrm{d}^4x\sqrt{-g}\left(R\Phi - \frac{\omega}{\Phi}g^{ab}\nabla_a\Phi\nabla_b\Phi + 16\pi L_m\right), \tag{1}$$

where R is the Ricci scalar and L_m is the matter Lagrangian. The field equations in the JF are $G_{ab} = S_{ab} + 8\pi T_{ab}/\Phi$, where T_{ab} is the matter energy-momentum tensor, G_{ab} is the Einstein tensor, and S_{ab} is given by

$$\begin{aligned} S_{ab} = {}& \frac{1}{\Phi}(\nabla_a\nabla_b\Phi - g_{ab}\nabla^c\nabla_c\Phi) + \frac{\omega}{\Phi^2} \\ & (\nabla_a\Phi\nabla_b\Phi - \frac{1}{2}g_{ab}g^{cd}\nabla_c\Phi\nabla_d\Phi). \end{aligned} \tag{2}$$

To rewrite the action and the resulting field equations in the Einstein frame (EF), in which the metric is $\widetilde{g}_{ab}$, one makes the field redefinition $\mathrm{d}\varphi = \mathrm{d}\Phi\sqrt{2\omega+3}/(2\Phi)$, which determines the relationship between ω and φ. One also needs to define the relationship between Φ and φ, and we adopt the notation $\mathcal{A}(\varphi) := 1/\sqrt{\Phi}$. The conformal transformation between the EF and JF metric is then $\widetilde{g}_{ab} = \mathcal{A}^{-2}g_{ab}$. In the EF, the ST theory is determined by the functional form of $\mathcal{A}(\varphi)$.

We are primarily concerned with analysing the internal structure of static, spherically symmetric neutron star solutions in the JF and, in particular, looking for violations of the weak energy condition (WEC) in this frame. However, it is easier to solve the field equations in the EF when scalarization effects are present. In addition, the general vacuum spherically symmetric ST solution is known in the EF (Coquereaux & Esposito-Farese 1990) and we need to match our solutions to a vacuum exterior, both to fix the value of the central scalar field and to find the solutions' masses. Hence we solve the EF equations of structure here, details of which can be found in the work by Damour and Esposito-Farese (1993).

To characterise the degree to which the scalar field is inhomogeneous, a convenient parameter is the scalar charge Q_S which, in the Jordan frame and for a spherically symmetric solution, is defined to be

$$Q_S := \lim_{r\to\infty}\left(r^2\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right), \tag{3}$$

where r is the Schwarzschild radial coordinate. This quantity is used to evaluate the conserved JF energy of the star,

$$M_T := M_{ADM} - \frac{1}{2}Q_S \tag{4}$$

where M_{ADM} is the ADM mass in the JF (Lee 1974). In the EF, the scalar charge $\widetilde{Q}_S$ associated with φ is defined in a analogous way. The EF scalar charge may be used to define an effective coupling strength $\alpha := \widetilde{Q}_S/\widetilde{M}_{ADM}$, where $\widetilde{M}_{ADM}$ is the ADM mass in the EF. This latter quantity is identical to the JF quantity M_T up to a conformal factor which is close to unity. In terms of JF quantities, the effective coupling α may be rewritten as $\alpha = (1/2)\sqrt{2\omega_B+3}\,Q_S/M_T$. Hence the EF quantity α is also a good indicator of scalar field inhomogeneity in the JF.

3. Energy conditions

In GR, the strong, dominant, and weak energy conditions are usually formulated by placing restrictions on the properties of the matter in a solution (see, for example, Hawking & Ellis 1973). By virtue of the GR field equations $G_{ab} = 8\pi T_{ab}$, any condition placed on T_{ab} is automatically satisfied by G_{ab}. Thus, in GR, the energy conditions are also statements about the geometry of any given solution. The singularity theorems, for example, rely on the behaviour of G_{ab} and make no explicit reference to the matter content of the theory. In ST gravity, the situation is a little more complex. The field equations allow both G_{ab} and T_{ab} to obey different conditions in the same solution. In particular, in a solution in which the normal matter obeys all three energy conditions, G_{ab} need not obey any. This is because S_{ab}, defined by Eq. (2), contains terms that are linear in the second derivatives of Φ, and no

restriction on the sign of these derivatives apply (see Sokołowski 1989a,b; Cho 1992; Magnano & Sokołowski 1994, Torres 2002 for discussions). Hence one can have a solution in which the normal, bosonic or fermionic, matter has a positive energy density and yet the WEC is violated. This would have implications for, for example, singularity theorems and other statements about the allowed global structure of a spacetime (see Anchordoqui et al. 1996 for an example, and Barcelo & Visser 2000 for further discussion). Hence it is reasonable to investigate whether spontaneous scalarization is accompanied by a spontaneous violation of WEC.

The WEC holds when the quantity $G_{ab}K^aK^b$ is non-negative for all timelike and null vectors K^a. In the JF, a local effective energy density $8\pi\mu := G_{ab}U^aU^b$ may be defined, where U^a is a unit timelike vector. This quantity is not a true energy density, since it mixes up contributions from both the matter and the Φ field. However, it is a useful quantity since a sufficient condition for WEC violation is $\mu < 0$. Since we are concerned here with static solutions, we shall take U^a to be the timelike unit Killing vector field. In terms of the metric and scalar fields, using the wave equation for Φ one can show that

$$\begin{aligned} 8\pi\mu = {} & \frac{1}{\Phi}U^aU^b\nabla_a\nabla_b\Phi + \frac{\omega}{2\Phi^2}\, g^{ab}\nabla_a\Phi\nabla_b\Phi \\ & +\frac{1}{\Phi}\left(8\pi\rho + \frac{8\pi(3p-\rho)}{2\omega+3} - \frac{1}{2\omega+3}\nabla^a\Phi\nabla_a\Phi\,\frac{\mathrm{d}\omega}{\mathrm{d}\Phi}\right), \end{aligned} \tag{5}$$

where we have used the assumption that T_{ab} is that of a perfect fluid. Integrating this quantity over a spacelike hypersurface orthogonal to U^a gives the ADM mass in the JF.

4. Neutron star solutions in ST gravity

As an example, we shall examine neutron star solutions in the two ST theories considered by Damour & Esposito-Farese (1993). The first theory is specified by the exponential function $\mathcal{A}(\varphi) = \mathrm{e}^{-\kappa\varphi^2}$, for which $\widetilde{\beta}_0 = -2\kappa$. This is equivalent to the JF theory with $2\omega + 3 = 1/(2\kappa\log\Phi)$. The second theory we consider is specified by the function $\mathcal{A}(\varphi) = \cos(\sqrt{\lambda}\,\varphi)$, for which $\widetilde{\beta}_0 = -\lambda$. This is equivalent to the JF theory with $2\omega + 3 = 1/[\lambda(\Phi - 1)]$. For both of these theories, the PPN constraint on γ is the most restrictive.

The above forms of the function $\omega(\Phi)$ diverge as Φ_B approaches the GR limit. For this reason, the JF descriptions of these theories break down and it is more appropriate to use field variables of a JF formalism developed by Damour & Esposito-Farese (1992, 1996) and in which the results of Damour & Esposito-Farese (1993, 1997) were presented.

We have integrated the equations of structure in the EF for neutron stars with the polytropic equation of state (EOS)

$$\rho = nm + \frac{Kn_0 m}{\Gamma - 1}\left(\frac{n}{n_0}\right)^{\Gamma}, \quad p = Kn_0 m\left(\frac{n}{n_0}\right)^{\Gamma}, \tag{6}$$

where m is the neutron mass, $n_0 = 1.0 \times 10^{44}\,\mathrm{m}^{-3}$ and the parameters K and Γ have the values $K = 0.0195$, $\Gamma = 2.34$. This EOS was used by Damour & Esposito-Farese (1993) as a best fit to a realistic description of high density neutron matter given by Diaz-Alonso & Ibanez-Cabanell (1985). We use the maximum values of φ_B consistent with the above PPN constraints (which are tighter than those used by Damour & Esposito-Farese 1993). For each solution, we compute effective density profiles in the JF, using Equation (5).

Fig. 1 (top panels) shows curves of α against baryonic mass M for solutions in both ST theories described above. The left panel is for the exponential theory $\mathcal{A}(\varphi) = \mathrm{e}^{-\kappa\varphi^2}$, while the right is for the cosine theory $\mathcal{A}(\varphi) = \cos(\sqrt{\lambda}\,\varphi)$. Aside from the lower values of φ_B, the curves are similar to those shown by Damour and Esposito-Farese (1993). However, they show an additional feature not reported in previous work in that some of the solutions, denoted by the dotted segments on some of the curves, contain a region in which the effective density μ is negative. Hence, spontaneous scalarization appears to be linked with a second phenomenon, that of spontaneous WEC violation.

As Fig. 1 (top panels) shows, only large mass stars which are already showing the scalarization effect exhibit WEC violation. The reason for this is as follows. From Equation (4), the energy of a star depends upon both M_{ADM} (which can be written as an integral of μ) and Q_S. This latter quantity is negative for all solutions, so tends to increase the star's energy. For a weak field star, φ and hence Φ are nearly homogeneous, Q_S is vanishingly small and μ is dominated by the matter density ρ. At the onset of scalarization, the increase in M_T due to a non-zero Q_S is cancelled by the decrease in M_{ADM} caused by the non-zero gradients in Φ reducing μ over much of the star's interior. Once this happens, an inhomogeneous Φ becomes energetically favoured and in some cases this causes μ to be negative in some regions of the star.

The existence of WEC violation also depends strongly on the chosen value of $\widetilde{\beta}_0$. For the exponential theory, WEC violation occurs when $\kappa \geq 3.0$, corresponding to $\widetilde{\beta}_0 < -6.0$. For the cosine theory, WEC violation occurs when $\lambda \geq 5.35$, corresponding to $\widetilde{\beta}_0 \leq -5.35$. It is apparent from these results that, for the same constraints on φ_B, different theories can exhibit WEC violation at different values of $\widetilde{\beta}_0$ and there is no reason to believe that other choices of $\mathcal{A}(\varphi)$ would not show negative energy density at larger values of $\widetilde{\beta}_0$.

Finally, we have found that larger values of φ_B decrease the value of $\widetilde{\beta}_0$ at which WEC

violation occurs. For example, by increasing the value of φ_B by a factor of approximately 15 (corresponding to an increase of Φ_B by less than 10%), we have found that negative density a region appears in stars in the exponential theory when $\kappa \geq 2$, corresponding to $\widetilde{\beta}_0 \leq -4$.

Damour and Esposito-Farese (1997) have found that ST theories must satisfy the constraint $\widetilde{\beta}_0 > -5$. This bound is based on a study of the binary pulsar in the exponential theory and is approximate, so could conceivably be slightly lower than reported. We have found that the cosine theory exhibits WEC violation at a value of $\widetilde{\beta}_0$ which is greater than that for the exponential coupling and closer to the inferred bound. There is no reason why other ST theories might not show spontaneous WEC violation at values of $\widetilde{\beta}_0 > -5$. Even if the bounds on β_0 were to exclude WEC violating solutions in this or other ST theories, one could still see this effect in real neutron stars: we have assumed here that each star is isolated and has a φ field which matches to the cosmological background field φ_B. However, a neutron star in the region of, for example, a strongly gravitating companion would be subject to different boundary conditions on its internal scalar field and, as we have discussed, this may allow WEC violation at a larger value of $\widetilde{\beta}_0$.

To show where within a neutron star WEC-violating regions occur, we have computed density profiles for several of the solutions shown in Fig. 1 (top panels). The bottom panels of Fig. 1 show the JF energy density as a function of Schwarzschild radius for several solutions containing WEC violating regions. Each curve is labelled by its baryonic mass and each curve terminates at the surface of the star. At this point, the density is dominated by the scalar field and is always positive. The dotted potion of each curve shows where the WEC is violated. The plot on the left shows solutions in the exponential theory and all are for $\kappa = 3.2$. The WEC violating solutions fall into two types. For the low mass solutions, the negative energy density region begins at the centre and extends part of the way out towards the surface. As the baryonic mass increases, the star's central density eventually becomes positive and the WEC violating region encloses a positive density core, as in the lower κ case. The primary reason for this is that, for larger mass solutions, the central matter density is very large and dominates over the WEC violating terms in S_{ab}. Only away from the star's centre, when the matter density falls, can the negative scalar field energy density start to dominate. The plot on the right of Fig. 1 (bottom panels) shows density plots for the cosine theory and all are for $\lambda = 6.0$. These are similar to those for the exponential theory. Density profiles for other values of $\widetilde{\beta}_0$ are similar.

5. Concluding Remarks

We have shown that spontaneous scalarization is accompanied by a second effect, that of spontaneous weak energy condition violation. This effect occurs for ST theories which give positive effective energy densities for matter solutions in a weak field region. It is only within a strong field regime, in compact objects, that the effect occurs. It is important to note that this is an effect limited in extent to inner regions of neutron stars. A distant observer will not see a negative energy or negative Kepler mass object (at least not according to our present, static, simulations) so it would not be possible to use the techniques outlined by Safonova et al. (2002) to observe these objects. There certainly could be, however, other observable effects in the evolution of the neutron star itself.

All in all, although we have shown that a mechanism for energy conditions violation is linked to a plausible evolutionary process (i.e. scalarization) in a known astrophysical environment (i.e. neutron stars), the stability of the latter under a scalarization process remains to be studied dynamically. It might be the case that no neutron stars are stable if scalarization occurs (due to the energy condition violation herein discovered); this would imply strong constraints on the plausibility of scalar-tensor theories to represent the observed Universe.

A natural extension of this work would be to determine whether and how a star can evolve from a positive energy density state to one which violates the WEC: our present study examines only static solutions. Three possible routes seem likely. The first is through accretion of matter. One can envisage a neutron star, whose baryonic mass is just below that needed for WEC violation, accreting matter to raise its mass to above the critical value. Although we do not have details of the time evolution of this process, it is reasonable to assume that the initial and final states correspond to a static solution similar to those discussed here. A second possibility is through gravitational collapse. For the ST theories we consider here, one needs strong gravitational fields to produce a Φ field that is inhomogeneous enough to cause WEC violation. Hence the interior of a massive star just before core collapse and neutron star formation would have a constant scalar field and a positive energy density. Once the neutron stars has been formed, the central density would then be large enough for WEC violation to occur. A third possibility is through gravitational evolution (as explored by, e.g., Torres 1997; Torres et al. 1998a,b; Whinnett & Torres 1999 for boson stars). In general, the background field φ_B (and its JF equivalent Φ_B) should increase as the Universe expands. Assuming that the star evolves adiabatically and can be modelled as a sequence of static equilibrium solutions, there may come a point in the star's lifetime when φ_B is large enough for it to suddenly acquire a WEC violating region. This possibility will be explored elsewhere.

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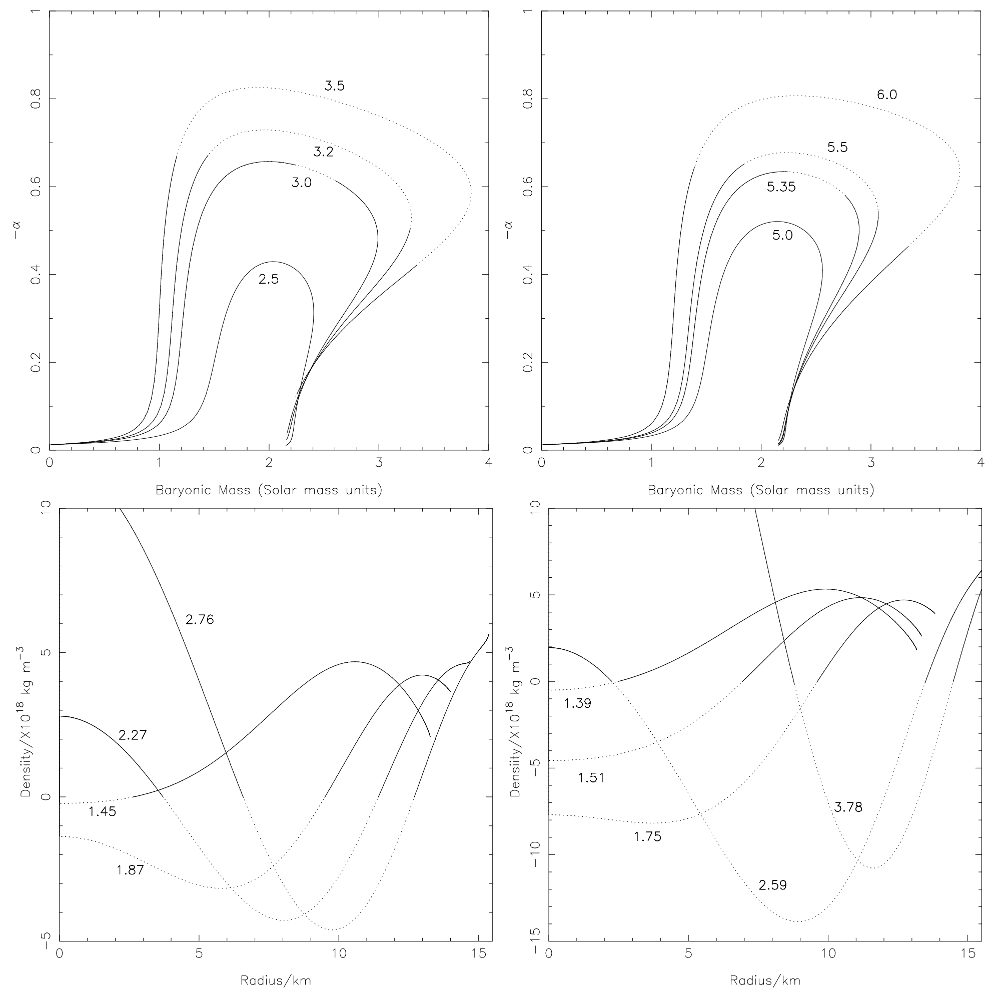

Fig. 1.— Top: Mass against scalar coupling for neutrons stars in the theory with $\mathcal{A}(\phi) = \mathrm{e}^{-\kappa\varphi^2}$ (left) and $\mathcal{A}(\phi) = \cos(\sqrt{\lambda}\,\varphi)$ (right). Bottom: Density against radius for neutron stars in theory with $\mathcal{A}(\phi) = \mathrm{e}^{-\kappa\varphi^2}$ (left) and $\mathcal{A}(\phi) = \cos(\sqrt{\lambda}\,\varphi)$ (right).